Minimum Weighted Vertex Cover on Difference Graphs and It's Algorithm

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Abstract:
Let μ(G) and γ(G) be vertex covering number and independent number of G. In this paper I compute minimum vertex cover on weighted vertex different graphs G_1D G_2 . An effective algorithm is illustrated to find minimum weighted vertex cover. The Independent Number of G_1D G_2 introduced.

Keywords: vertex cover, Algorithm, Independent number.

1. Introduction

Let G(V,E) undirected graph with weighted vertices, it's weight function W: V→ R, the set N⊂V is minimum weighted vertex cover of G. Among all covers of μ has smallest weight such that $\sum_{v \in V} W(V)$ is minimum.
The most famous algorithms used to solve this problem is genetic algorithms, neural networks, ant colony algorithms and local search algorithms. Despite the advantages of heuristic methods, it may have some problems such as dropping in a local minimum or it cannot reach an optimum solution.
Other solve the problem. Some of them based on assumption weights of the vertices [8]. Other algorithms calculate the degree of the vertex according their adjacent vertices [9]. Details of approximations for solving the problems is referred in references, an independent set or stable set, is a set of vertices in a graph where no two of vertices are adjacent. A maximum independent set is an independent set of largest size for graph G. Several algorithms were developed to calculate this set which is considered as NP hard problem as well. The minimum vertex cover is corresponding to the complement of the maximum independent set. Several algorithms were introduced to solve the minimum set cover based on this relation. The constant difference in all pairs of consecutive or successive numbers in a sequence is called the common difference, denoted by the letter "d ". It may be positive or negative according to whether the sequence is increasing or decreasing.

Methodology
Research Design
The study utilizes a qualitative methodology through a systematic review design to gather, evaluate, and synthesize existing research on the Minimum Weighted Vertex Cover On Different Graphs . The systematic review follows a structured process to ensure comprehensive topic coverage and minimize bias.
Literature Search Strategy
The literature search is conducted using a variety of academic databases and search engines, including JSTOR , Google Scholar, IEEE Xplore, and other relevant sources. The articles included in this paper were published in a period which the theoretical frameworks

,computational methods, theoretical frameworks and applications of graphs have developed widely.

2. Main results

The difference graph $G_1 D G_2$ of connected vertex weighted graphs G_1 , G_2 has vertex set $WV(G_1DG_2) = W(V(G_1) = WV(G_2)$, and set of edges $E(G_1DG_2) = [E(G_1) – E(G_2)] \cup [E(G_2) – E(G_1)]$.

Minimum vertex covering no. of G(μ W(G)) is minimum set of vertices covering all edges of G.

An important proposition about graph different:

Proposition 2.1: Any graph different of connected graphs must be connected.

Lemma 2.2:

Let $G_1 \cap G_2 = M$, $(G_1DG_1) = (G_1 – E(M)) \cup (G_2 – E(M)) = M_1^C \cup M_2^C$ for each M_i^C then : $\mu(M_i^C) = \mu(G_i) – \max\{O_i\}$ where $\max\{O_i\}$ is maximum cardinal number $\{ u \in WV(M) - c_I = \emptyset\}$, c_i minimum vertex covering set, i=1,2.

Theorem 2.3:

Let $G_1 \cap G_2 = M$, $G_1DG_2 = (G_1 – E(M)) \cup (G_2 – E(M)) = M_1^C \cup M_2^C$ then:

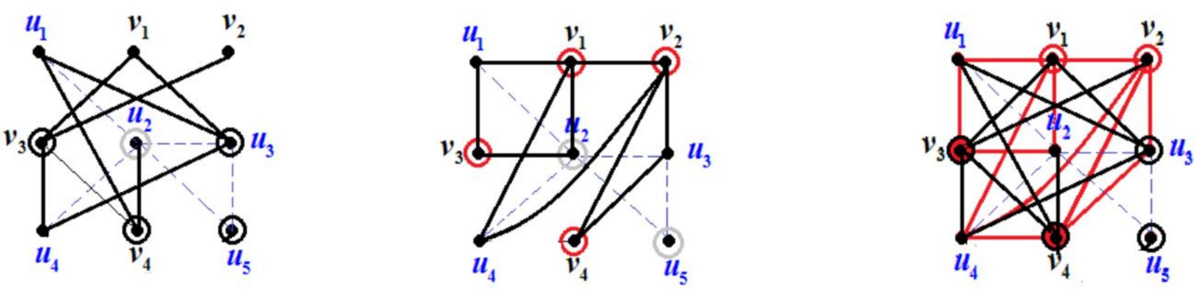
$\mu(G_1DG_2) \leq \mu(G_1) \cup \mu(G_2)$.

Proof:

Since $G_1 D G_2 = (G_1 – E(M)) \cup (G_2 – E(M)) = M_1^C \cup M_2^C$ then

$\mu(G_1DG_2) = \mu(G_1^C) \cup \mu(G_2^C) \leq \mu(G_1^C) + \mu(G_2^C)$.

Example 2.4:



G_1 G_2 G_1DG_2

Figure (1).

Let G_1 and G_2 are two weighted vertices graphs as shown in figure (1) , and weights of vertices illustrated in table 1:

u_1	u_2	u_3	u_4	u_5	v_1	v_2	v_3	v_4
6	7	1	6	2	5	7	2	2

Table 1

We can compute the follows:

Minimum vertex cover of G_1 : U_2 , U_3 , V_3 = 7+1+2=10 .

Minimum vertex cover of G_2 : U_2 , V_2 , U_1 , V_4 = 7+3+6+2=18 .

Minimum vertex cover of $G_1 D G_2$: U_3 , V_1 , V_3 , V_4 = 1+5+2+3=11 .

Lemma 2.5:

For connected vertex weighted graph G with order n , $\mu(G) + \gamma(G) = n$.

Theorem 2.6:

If $G_1 \cap G_2 = M$, $G_1 D G_2 = [G_1 - E(M)] \cup [G_2 - E(M)] = M_1 \cup M_2$, then $\mu(G_1 D G_2) = \mu(G_1) \cup \mu(G_2) - (|Q_1| + |Q_2| + |R|)$.

Where R = { u | u ∈ $minimum\ vertex\ cover\ M_1{}^c$ and $M_2{}^c$, $\max\{Q_i\}$ equal to the maximum cardinal number of Q_i.

Proof:

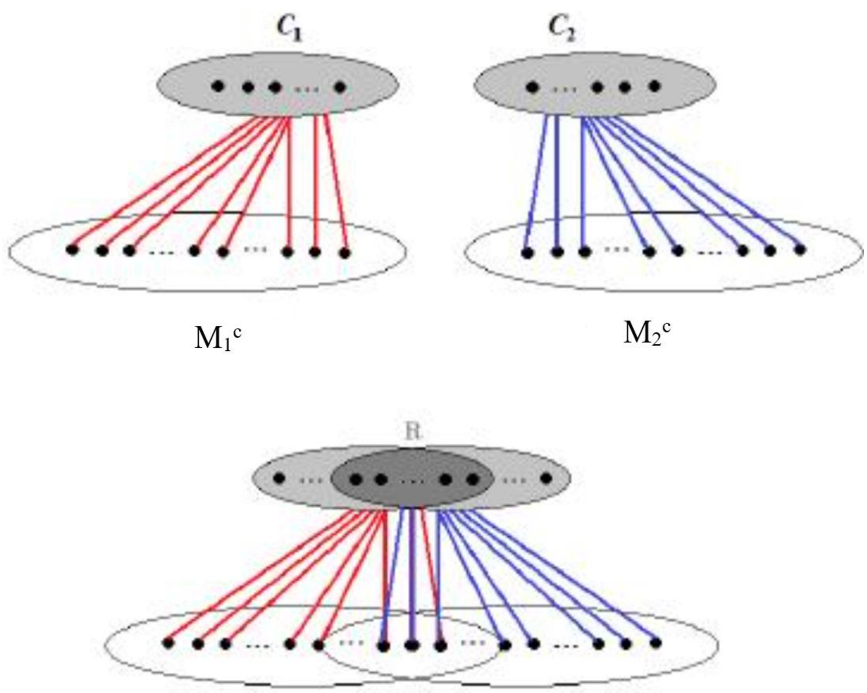
$G_1 D G_2 = [G_1 - E(M)] \cup [G_2 - E(M)] = M_1 \cup M_2$, we get $\mu(G_1 D G_2) =$

$\mu(M_1 \cup M_2) \leq \mu(M_1) + \gamma(M_2)$.

Let R = {u|u ∈ the minimum vertex covering set of $M_1$and M_2}, then by definition of difference graph, $E(M_1) \cap E(M_2) = \emptyset$, and:

$\gamma(G_1 D G_2) = \gamma(M_1) + \gamma(M_2) - |R|$.

$\gamma(G_1 D G_2) = \gamma(G_1) \cup \gamma(G_2) - (|Q_1| + |Q_2| + |R|)$.



$G_1 D G_2 = M_1{}^c \cup M_2{}^c$

Figure (2)

3. The Algorithm:

We can find out minimum weighted vertex cover of difference graph by the following algorithm:

INPUT: Difference Weighted vertex graph G.

OUTPUT: The minimum vertex cover of G.

1. While e $\in E = \emptyset \quad do$

2. Start with V_i which have min(weight $w(V_i)$ $\forall\, i, j = 1,2, \ldots \ldots n$

3. If $W(V_i) = W(V_j)$ then

4. Select either V_i or V_j

5. $\text{Weight}(V_i) = \text{Weight}(V_i, V_j)$ – no. of connected adjacent $W(V_i, V_j)$

6. End if

7. End While.

Lemma 3.1:

$\forall M_i^{\backslash}$, then : If $G_1 \cap G_2 = M$ and $G_1 D G_2 = (G_1 - E(M) \cup [G_2 - E(M)] = M_1^{\backslash} \cup M_2^{\backslash}$

$\mu(M_i^{\backslash}) = \mu(G_i)$ – maximum{ O_i }, where maximum{ O_i } is cardinal number $O_i = \{ v \in V(M) \mid N_{Mi}(v) - K_i = \phi \}$, K_i minimum vertex covering set of G_i I =1, 2.

Proof:

For $M_i^{\backslash} = (G_i - E(M))$, $\mu(M_i^{\backslash}) = \mu(G_i - E(M)) \leq \mu(G_i)$.

Let $O_i = \{v \in V(M) \mid N_{Mi}(v) - K_i = \phi\}$

Choosing minimum vertex covering set K_i of $G_i \ni O_i$ have max. cardinal number O_i, then minimum vertex covering of $M_i^{\backslash}$ have

$\mu(M_i^{\backslash}) = \mu(G_i) - \max\{O_i\}$, i = 1,2.

4. Independent Number of $G_1 D\, G_2$:

For graph G , The cardinality of the largest independent set of vertices in G is called the independence number of G and is denoted by $\mu(G)$.

Theorem 4.1:

If $G_1 \cap G_2 = H$, $G_1 D G_2 = [G_1 - E(M)] \cup [G_2 - E(M)] = M_1 \cup M_2$. For each M_i, then $\mu(G_1 D\, G_2) = M_1^C + M_2^C - |S|$, where $S = \{r \mid r \in$ the maximum independent set of $M_1, M_2\}$.

Proof:

$\mu(G_1 D\, G_2) + \gamma(G_1 D\, G_2) = n$ $\mu(G_1 D\, G_2) = n - \gamma(M_1) - \gamma(M_2) + |R| = n - (n - \mu(M_1)) - (n - \gamma(M_2)) + |R| = \mu(M_1) + \mu(M_2) - (n - |R|) = \mu(M_1) + \mu(M_2) - |S|$.

lemma 4.2:

the number of vertices of a graph is the same as its minimum vertex cover number added to the size of a maximum independent set.

Proof:

By removing all vertices of minimum vertex cover leads to maximum independent set.

So if S is the size of minimum vertex cover of G(V,E) then the size of maximum independent set of G is |V| – S.

Conclusion:

It was established that Minimum vertex covering no. of G(μ W(G)) is minimum set of vertices covering all edges in G.

On the basis of the conducted research, the algorithm of minimum weighted vertex cover of difference graph obtained . the proposed algorithm can be considered as one of the exact algorithms benefiting with the properties of arithmetic sequence and character encoding of graphs.

Acknowledgment:

The authors would like to thank the Deanship of Scientific Research at Umm Al-Qura University for supporting this work by Grant Code: (23UQU4331140DSR03).

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